King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 254: Discrete Structures II
Fall semester 2016-2017 (161)
Major Exam #2, Wednesday November 30, 2016
Time: **120** Minutes

Name: _____

ID#: _____

Instructions:
1. The exam consists of 8 pages, including this page, containing 6 questions.
2. Answer all questions. **Show all the steps.**
3. Make sure your answers are **clear** and **readable**.
4. The exam is closed book and closed notes. **No calculators** or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
5. If there is no space on the front of the page, use the back of the page.

Question	Maximum Points	Earned Points
1	15	
2	15	
3	15	
4	20	
5	25	
6	10	
Total	**100**	

***. Some Useful Formulas:**

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$ $\sum_{i=1}^{n} i.c^i = \Theta(1)$ for $0 < c < 1$ $2^{\lg n} = n$,

$log_b a = \frac{log_c a}{log_c b}$ where $c, b \neq 1$ $\log a^b = b \log a$ $\log ab = \log a + \log b$

Q1: **[15 points] Answer the following Questions.**

a) [4 points] The first nine digits of the ISBN-10 of a certain book are 013850587. Compute the check digit for that book?

b) [4 points] Encrypt the message CHEAR UP by translating the letters into numbers, applying the encryption function $f(p) = 17p + 22 \pmod{26}$, and then translating the numbers back into letters.

c) [4 points] Decrypt the message LO WI PBSOXN encrypted using the shift cipher

$$f(p) = 17p + 22 \pmod{26}$$

d) [3 points] In RSA Cryptosystem, the knowledge of the values (n, e) used in the encryption process is known to everyone.

i. (1 points) How come it is highly unlikely for others to be able to decrypt the message.

ii. (1 points) What does the receiver of the encrypted message need to know/have in order to decrypt the message?

iii. (1 points) Describe the decryption process.

Q2: [15 points]

Consider the following relations defined on the set of all *natural numbers*:

$R_1 = \{(a, b) \in \mathbb{N}^2 \mid a > b\}$,
$R_2 = \{(a, b) \in \mathbb{N}^2 \mid a \geq b\}$,
$R_3 = \{(a, b) \in \mathbb{N}^2 \mid a < b\}$,
$R_4 = \{(a, b) \in \mathbb{N}^2 \mid a \leq b\}$,
$R_5 = \{(a, b) \in \mathbb{N}^2 \mid a = b\}$ and
$R_6 = \{(a, b) \in \mathbb{N}^2 \mid a \neq b\}$.

Find the following relations:

i. $R_1 - R_2$
ii. $R_1 \cup R_3$
iii. $R_2 \oplus R_4$
iv. $R_5 \circ R_6$
v. $R_1 \circ R_4$

Q3: [15 points] Solve the following questions.

a) [3 points] How many nonzero entries does the matrix representing the relation R on $A = \{1, 2, 3, \ldots, 100\}$ consisting of the first 100 positive integers have if

$$R = \{(a, b) \mid a > b\}$$

b) [12 points] Consider the following relation $R = \{(a, c), (b, d), (c, a), (c, b), (d, b), (e, d)\}$ on $\{a, b, c, d, e\}$. Using the definition of the transitive closure, find M_{R^*} and then list the elements of the relation R^*.

Q4: [20 points]

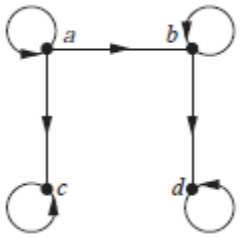
Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$.

a) [15 points] Show that R is an equivalence relation.

b) [5 points] What is the equivalence class of (1, 2) with respect to R?

Q5: [25 points]

a) [3 points] Determine whether the relation with the directed graph shown below is a partial order. Justify your answer.



b) [22 points] Answer these questions for the poset ({3, 5, 9, 15, 24, 45}, |).

i. (6 points) Draw the Hass diagram for the above poset.
ii. (2 points) Find the maximal elements.
iii. (2 points) Find the minimal elements.
iv. (1 points) Is there a greatest element?
v. (1 points) Is there a least element?
vi. (3 points) Find all upper bounds of {3, 5}.
vii. (2 points) Find the least upper bound of {3, 5}, if it exists.
viii. (3 points) Find all lower bounds of {15, 45}.
ix. (2 points) Find the greatest lower bound of {15, 45}, if it exists.

Q6: [10 points]

For which values of *n* are these graphs bipartite? Briefly justify your answer.

a. Complete graphs K_n.
b. Cycles C_n.
c. Wheels W_n.
d. *n*-Cubes Q_n.